19.02.2026

Contraction mapping

Let (X, d) be a metric space. Let T be a mapping of X into itself.
Then T is called a contraction mapping if there exists a positive real number $r < 1$ with the property that

$$d(Tx, Ty) \leq r\, d(x, y) \quad \forall\, x, y \in X.$$

It is clear that application of T to each of the points x and y contracts the distance between them, ~~been~~ as the distance between images of any two points is less than the distance between the points.

Continuous mapping (at a point)

Let (X, d_1) and (Y, d_2) be two metric spaces. Let $f : X \to Y$.
Then f is said to be continuous at a point x_0 in X if either of the following conditions is satisfied:

(i) for each $\epsilon > 0$, $\exists \delta > 0$ such that

$$d_1(x, x_0) < \delta \Rightarrow d_2(f(x), f(x_0)) < \epsilon$$

(ii) for each open sphere $S_\epsilon(f(x_0))$ centred on $f(x_0)$, there exists an open sphere $S_\delta(x_0)$ centred on x_0 such that

$$f(S_\delta(x_0)) \subseteq S_\epsilon(f(x_0)).$$

<u>Continuous mapping ~~of~~ at a set</u>

Let (X, d_1) ~~be~~ and (Y, d_2) be two metric spaces. Let $f: X \to Y$ be a mapping from X into Y.

Then f is continuous iff

$$x_n \to x \Rightarrow f(x_n) \to f(x).$$